$$\sqrt[6]{\frac{1}{b^2(a^2-b^2)^2}}\cdot\sqrt[6]{\frac{(a+b)^3}{(a-b)^2}}=\sqrt[6]{\frac{1}{b^2(a+b)^2(a-b)^2}\cdot\frac{(a+b)^3}{(a-b)^2}}=\sqrt[6]{\frac{a+b}{b^2(a-b)^4}}.$$

$$\sqrt[3]{\frac{a(a-b)}{b(a+b)}}\cdot\sqrt[15]{\frac{(a+b)^2}{(a-b)^2}}:\sqrt[5]{\frac{a-b}{a+b}}=\sqrt[15]{\frac{a^5(a-b)^5}{b^5(a+b)^5}\cdot\frac{(a+b)^2}{(a-b)^2}\cdot\frac{(a+b)^3}{(a-b)^3}}=\sqrt[15]{\frac{a^5}{b^5}}=\sqrt[15]{\left(\frac{a}{b}\right)^5}=\sqrt[3]{\frac{a}{b}}.$$

Per $a^2-4<0$, cioè per $-2<a<2$, si ha:

$$\frac{a^2-4}{2}\cdot\sqrt[8]{\frac{256}{(a+2)^{10}(a-2)^6}}=-\sqrt[8]{\frac{(a^2-4)^8}{256}\cdot\frac{256}{(a+2)^{10}(a-2)^6}}=$$

$$=-\sqrt[8]{\frac{(a+2)^8(a-2)^8}{(a+2)^{10}(a-2)^6}}=-\sqrt[8]{\left(\frac{a-2}{a+2}\right)^2}=-\sqrt[4]{\left|\frac{a-2}{a+2}\right|}=-\sqrt[4]{\frac{2-a}{a+2}};$$

mentre per $a^2-4\geq 0$, cioè per $a\leq -2$, $a\geq 2$, si ha:

$$\frac{a^2-4}{2}\cdot\sqrt[8]{\frac{256}{(a+2)^{10}(a-2)^6}}=\sqrt[8]{\left(\frac{a-2}{a+2}\right)^2}=\sqrt[4]{\frac{a-2}{a+2}}.$$

$$\sqrt[3]{a^6(a+2)}-a\sqrt[3]{8(a+2)}+\sqrt[6]{(a+2)^2}=a^2\sqrt[3]{a+2}-2a\sqrt[3]{a+2}+\sqrt[3]{a+2}=$$

$$=(a^2-2a+1)\sqrt[3]{a+2}=(a-1)^2\sqrt[3]{a+2}.$$

Si noti che certamente è: $a-b>0$; $a+b>0$.
Si ha:

$$E=\sqrt[6]{\left(\sqrt{a^2+b^2}+\sqrt{2ab}\right)\cdot\left(\sqrt{a^2+b^2}-\sqrt{2ab}\right)}:\sqrt[3]{a+b}+2\sqrt[3]{\frac{a-b}{a+b}}=$$

$$=\sqrt[6]{\left(\sqrt{a^2+b^2}\right)^2-\left(\sqrt{2ab}\right)^2}:\sqrt[3]{a+b}+2\sqrt[3]{\frac{a-b}{a+b}}=\sqrt[6]{a^2+b^2-2ab}:\sqrt[3]{a+b}+2\sqrt[3]{\frac{a-b}{a+b}}=$$

$$=\sqrt[6]{(a-b)^2}:\sqrt[3]{a+b}+2\sqrt[3]{\frac{a-b}{a+b}}=\sqrt[3]{a-b}:\sqrt[3]{a+b}+2\sqrt[3]{\frac{a-b}{a+b}}=\sqrt[3]{\frac{a-b}{a+b}}+2\sqrt[3]{\frac{a-b}{a+b}}=3\sqrt[3]{\frac{a-b}{a+b}}.$$

Si osservi che è $x > 0$, $\dfrac{x}{a} > 0$ e quindi anche $a > 0$.
Quindi si ha:

$$\left(\frac{\sqrt[4]{a^4x}}{\sqrt[3]{a^2}} : \sqrt[3]{\sqrt{x^2 \cdot \frac{x}{a}}}\right) : \frac{1}{\sqrt[4]{x}} = \left(\frac{\sqrt[12]{a^{12}x^3}}{\sqrt[12]{a^8}} : \sqrt[6]{\frac{x^3}{a}}\right) : \frac{1}{\sqrt[4]{x}} = \left(\sqrt[12]{\frac{a^{12}x^3}{a^8}} : \sqrt[12]{\frac{x^6}{a^2}}\right) : \frac{1}{\sqrt[4]{x}} =$$

$$= \left(\sqrt[12]{a^4x^3} : \sqrt[12]{\frac{x^6}{a^2}}\right) : \frac{1}{\sqrt[4]{x}} = \sqrt[12]{a^4x^3 \cdot \frac{a^2}{x^6}} \cdot \sqrt[4]{x} = \sqrt[12]{\frac{a^6}{x^3}} \cdot \sqrt[4]{x} = \sqrt[4]{\frac{a^2}{x}} \cdot \sqrt[4]{x} = \sqrt[4]{\frac{a^2}{x} \cdot x} = \sqrt[4]{a^2} = \sqrt{a}.$$

$$\frac{b-1}{b+1}\sqrt[6]{\frac{b^2}{(b^3-3b^2+3b-1)^2}} + \frac{b+1}{b-1}\sqrt[3]{\frac{b}{b^3+3b^2+3b+1}} =$$

$$= \frac{b-1}{b+1} \cdot \frac{\sqrt[3]{b}}{|b-1|} + \frac{b+1}{b-1} \cdot \frac{\sqrt[3]{b}}{b+1} = \sqrt[3]{b}\left[\frac{b-1}{(b+1)|b-1|} + \frac{1}{b-1}\right].$$