ESERCIZIO1

a)

$y(x) := \frac{\sin(x)}{\sqrt{x}}$

$$x \rightarrow \frac{\sin(x)}{\sqrt{x}} \quad (1)$$

$diff(\, y(x)\, , x)$

$$\frac{\cos(x)}{\sqrt{x}} - \frac{1}{2}\,\frac{\sin(x)}{x^{3/2}} \quad (2)$$

derivata prima $y' = \frac{2\,x\cos x - sen\,x}{2\,x\sqrt{x}}$

$diff(\, y(x)\, , [x\$2])$

$$-\frac{\sin(x)}{\sqrt{x}} - \frac{\cos(x)}{x^{3/2}} + \frac{3}{4}\,\frac{\sin(x)}{x^{5/2}} \quad (3)$$

derivata seconda $y'' = \frac{(3-4\,x^2)\sqrt{x}\,sen\,x - 4\,x\sqrt{x}\cos x}{4\,x^3}$

b)

$f(x) := x^{-3} + \sqrt[5]{x^3}$

$$x \rightarrow \frac{1}{x^3} + (x^3)^{1/5} \quad (4)$$

$diff(\, f(x)\, , x)$

$$-\frac{3}{x^4} + \frac{3}{5}\,\frac{x^2}{(x^3)^{4/5}} \quad (5)$$

derivata prima $f' = -\frac{3}{x^4} + \frac{3}{5}x^{\frac{2}{5}}$

$diff(\, f(x)\, , [x\$2])$

$$\frac{12}{x^5} - \frac{36}{25}\,\frac{x^4}{(x^3)^{9/5}} + \frac{6}{5}\,\frac{x}{(x^3)^{4/5}} \quad (6)$$

derivata seconda $f'' = \frac{12}{x^5} + \frac{6}{5}\,x^{-\frac{7}{5}} = \frac{12}{x^5} + \frac{6}{5\sqrt[5]{x^7}}$

c)

$h(x) := \dfrac{x^3+4\cdot x^2-3\cdot x}{x^4-5\cdot x+8}$

$$x \to \frac{x^3+4x^2-3x}{x^4-5x+8} \tag{7}$$

diff$(\ h(x)\ ,x)$

$$\frac{3x^2+8x-3}{x^4-5x+8} - \frac{(x^3+4x^2-3x)\,(4x^3-5)}{(x^4-5x+8)^2} \tag{8}$$

derivata prima $\ h' = \dfrac{-x^6+4x^5+9x^4-10x^3+4x^2+64\cdot x-24}{(x^4-5x+8)^2}$

diff$(\ h(x)\ ,\ [x\$2])$

$$\frac{6x+8}{x^4-5x+8} - \frac{2\,(3x^2+8x-3)\,(4x^3-5)}{(x^4-5x+8)^2} + \frac{2\,(x^3+4x^2-3x)\,(4x^3-5)^2}{(x^4-5x+8)^3} - \frac{12\,(x^3+4x^2-3x)\,x^2}{(x^4-5x+8)^2} \tag{9}$$

derivata seconda $\ h''$

$= \dfrac{-6x^9+20x^8+36x^7+30x^6-140x^5+44x^4+288x^3-40x^2-256x+512}{(x^4-5x+8)^3}$

ESERCIZIO 2

$y := x^3+4\cdot x+\sin(x)$

$$x^3+4x+\sin(x) \tag{10}$$

derivata prima $\ $*diff*$(\ y\ ,x)$

$$3x^2+4+\cos(x) \tag{11}$$

deivata seconda $\ $*diff*$(y\ ,\ [x\$2\ ])$

$$6x-\sin(x) \tag{12}$$

derivata terza $\ $*diff*$(\ y\ ,\ [x\$3\ ])$

$$6-\cos(x) \tag{13}$$

valore decimale di $\ \dfrac{5}{6}\pi \qquad$ *evalf*$\left(\dfrac{5}{6}\pi\right)$

$$2.617993878 \tag{14}$$

valori della funzione e delle derivate in x = 5/6 π

evalf$(\ $*subs*$(\ x=2.62\ ,x^3+4\cdot x+\sin(x)\ )\ )$

$$28.96298964 \tag{15}$$

evalf$($*subs*$(x=2.62\ ,\ 3\cdot x^2+4+\cos(x)\ )\ )$

$$23.72617328 \tag{16}$$

evalf$($*subs*$(x=2.62\ ,\ 6\cdot x-\sin(x)\ )\ )$

$$15.22173836 \tag{17}$$

evalf$($*subs*$(x=2.62\ ,\ 6-\cos(x)\ )\ )$

$$6.867026721 \quad (18)$$

POLINOMIO DI TAYLOR RICHIESTO

$$y\ (\ 2.62\ +h) = 28,96\ +23,73\ h + \frac{15.22}{2}h^3 + \frac{6.87}{6}h^3$$

$y := \cos(x) + \sin(2\cdot x)$

$$\cos(x) + \sin(2\,x) \quad (19)$$

$diff(\ y, x)$

$$-\sin(x) + 2\cos(2\,x) \quad (20)$$

$diff(\ y, [x\$2])$

$$-\cos(x) - 4\sin(2\,x) \quad (21)$$

$diff(\ y, [x\$3])$

$$\sin(x) - 8\cos(2\,x) \quad (22)$$

$evalf\left(\frac{\pi}{5}\right)$

$$0.6283185308 \quad (23)$$

valori della funzione e delle derivate in x = 0.63

$evalf(subs(\ x=0.63\ , y))$

$$1.760117850 \quad (24)$$

$evalf(subs(x=0.63\ , -\sin(x) + 2\cos(2\,x)))$

$$0.0224890589 \quad (25)$$

$evalf(subs(x=0.63\ , -\cos(x) - 4\sin(2\,x)))$

$$-4.616388874 \quad (26)$$

$evalf(subs(x=0.63\ , \sin(x) - 8\cos(2\,x)))$

$$-1.857390509 \quad (27)$$

POLINOMIO DI TAYLOR

$$y(0.63\ +h) = 1,76 + 0,02\ h - \frac{4.62}{2}h^2 - \frac{1.86}{6}h^3$$

ES 3

$$\mathrm{D}\left(f^2 + 5\,g\right) = 2\ f\ Df\ + 5\,Dg\ = 2\ (x+5)\cdot 1 + 5\left(\cos x + e^x\right) = 2\,x + 5\left(\ 2 + \cos x + e^x\right)$$

$$\mathrm{D}\left(\frac{f}{g} + \frac{g^2}{x^3}\right) = \frac{f'g - g'f}{g^2} + \frac{2\,g\,g'\,x^3 - 3\,x^2g^2}{x^6} = \frac{\left(sen x + e^x\right) - \left(\cos x + e^x\right)(x+5)}{\left(sen\,x + e^x\right)^2} + \frac{2\left(sen\,x + e^x\right)\left(\cos x + e^x\right)x^3 - 3\,x^2\left(sen\,x + e^x\right)^2}{x^6}$$

ricerca delle tangenti

$g(x) := \sin(x) + e^x$

$$x \to \sin(x) + e^x \quad \textbf{(28)}$$

$f(x) := x + 5$

$$x \to x + 5 \quad \textbf{(29)}$$

$diff(\, g(x)\,, x)$

$$\cos(x) + e^x \ln(e) \quad \textbf{(30)}$$

coordinate del punto $\left(\frac{3}{4}\pi\ ,\ \frac{\sqrt{2}}{2} + e^{\frac{3}{4}\pi}\right)$

valore coeff angolare retta tangente $-\frac{\sqrt{2}}{2} + e^{\frac{3}{4}\pi}$

retta tangente $y - \frac{\sqrt{2}}{2} + e^{\frac{3}{4}\pi} = \left(-\frac{\sqrt{2}}{2} + e^{\frac{3}{4}\pi}\right)\left(x - \frac{3}{4}\pi\right)$

$t(x) := \frac{f(x)}{g(x)}$

$$x \to \frac{f(x)}{g(x)} \quad \textbf{(31)}$$

derivata prima $t' = \frac{\left(\sin(x) + e^x\right) - (x+5)\left(\cos(x) + e^x\right)}{\left(\sin(x) + e^x\right)^2}$

coordinate del punto di tangenza (0,29 ; 9,74)

coefficiente angolare $t'\,(\,0.29\,) = -6{,}85$

Equazione retta tangente $y - 9{,}74 = -6{,}85\ (\,x - 0{,}29)$